**ECOM90024**

**FORECASTING IN ECONOMICS AND BUSINESS**

**PRACTICE EXAM**

**SEMESTER 1, 2023**

**NOTE: PLEASE COMPUTE ALL OF YOUR NUMERICAL ANSWERS TO 3 DECIMAL PLACES**

**Question 1 (11 Marks)**

Let be governed by the following process,

The parameters , , , are unknown but you have observed a set of realizations .

1. **(5 Marks)** Outline the steps that you would take in order to test whether there exists a unit root in the data generating process of . Be as explicit and precise as possible.
2. **(2 Marks)** Using words, explain how your forecast intervals of will depend on the outcome of the procedure that you outlined in part (a).
3. **(4 Marks)** Assume that is trend stationary. Describe two appropriate approaches to estimating the parameters and . Be as explicit and precise as possible.

**Question 2 (10 Marks)**

Let for be a time series that behaves according to the following equation,

1. **(2 Marks)** Compute the unconditional mean and variance of .

1. **(2 Marks)** Is a covariance stationary process? Please make sure to include your reasoning and any derivations or computations that you have employed.
2. **(2 Marks)** Is an invertible process? Please make sure to include your reasoning and any derivations or computations that you have employed.
3. **(4 Marks)** Derive the autocorrelation function of and provide an appropriate visual depiction of it.

**Question 3 (17 Marks)**

Suppose that you have generated the following sample autocorrelation function (ACF) and sample partial autocorrelation function (PACF) from a set of time series observations

 

SAMPLE PACF

SAMPLE ACF

1. **(4 Marks)** Using the visual features depicted in the sample ACF and PACF and your knowledge of the stochastic properties of autoregressive and moving average models, argue that an AR(2) would be a more appropriate specification for compared to an MA(2).
2. **(4 Marks)** Suppose that the sample autocorrelations are computed as,

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  | 0.824 | 0.752 | 0.660 | 0.573 | 0.522 | 0.446 | 0.388 | 0.348 | 0.282 |

Assuming that the true data generating process is an AR(2), write down the expressions for the first two autocorrelations in terms of the AR(2) coefficients and and verify that their Yule-Walker estimates are given by and

1. **(2 Marks)** If the data generating process is an AR(2), its variance will be given by,

Now suppose that the sample variance of your time series is computed to be . Using this and the quantities that you verified in part (2b), verify that the estimate of the innovation variance is given by .

1. **(6 Marks)** Assume that the data generating process has an unconditional mean of 0 and suppose that the last two observations in the sample take values and . Using the quantities that you verified in parts (2b) and (2c), compute 1-step and 2-step ahead point and 95% interval forecasts.
2. **(1 Mark)** Briefly describe in words what will happen to the forecast interval as the forecast horizon becomes arbitrarily large, (i.e. ).